Evaluating Influence Diagrams

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Paper Reference Number: 60-11-30-102
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Abstract

Influence Diagrams (ID) are graphical representation for a problem scenario. These diagrams consist of arcs and nodes. Contrasting Bayesian Networks (BN) which only uses chance nodes, ID includes other types of nodes as well. Decisions can be modeled easily using ID, but for having a correct output, the graphs should be evaluated first. There are various algorithms for valuation of these diagrams. Calculating the efficiency of the developed ID is also crucial. $\mathcal{E}=\hat{A}^{\alpha}.\ \hat{C}^{1-\alpha}$ is the efficiency function for an ID. The parameters involved are Accuracy (A), and Complexity (C). In addition, α is the value set by the decision maker. This paper would review existing evaluation algorithms. Moreover, the Influence Diagrams for London Plan Policy 4A will be evaluated by means of reviewed algorithms. Finally, the efficiency of the Policy 4A ID will be analyzed.

Key words: Influence Diagrams, London Plan, Reversal and Removal, Probability Expansion

1. Introduction

Since the introduction of Influence Diagrams in 70s, many evaluation techniques have been introduced. The reason for starter of Influence Diagrams was overcoming the shortcomings of decision trees (Qi & Poole, 1995). The first technique was conversion of the ID into a decision tree and then solving the decision tree using the available algorithms (Shenoy, 1994). Shachter proved that there are other means of directly evaluating an ID. Hence the 'Arc Reversal' and 'Variable Elimination' algorithms were introduced which are still in use (Shachter, 1986). There are various other algorithms introduced in recent years which are able to directly evaluate an Influence Diagram without converting it to any other network.

2. Evaluation Algorithms

If the paper is the result of a research, then the data and material used in the research should be presented here Arc reversals method is performed in way that the consistency between the mathematical and numerical probabilities with the actual nodes and relations remain the same. Any rules for arc reversals must be therefore rationale in terms of the calculus of conditional probabilities.

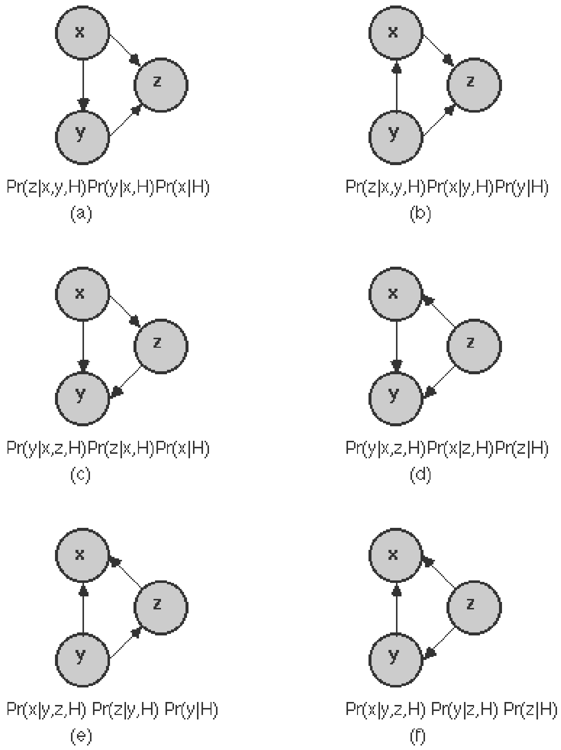

Fig. 1: Probability Expansion in an ID consisting of nodes 'x', 'y', and ‘z’ (Agogino, 1999)

An arc from state node "y" to state node "x" can be reversed if "y" is not a multi-path predecessor of "x". This is to disallow any arc reversal that may cause a cycle to be introduced in the diagram. A node "y" is called a multi-path predecessor of node "x" if it has more than one path to y. On reversal of an arc from y to x, node "y" inherits the direct predecessors of "x" and vice-versa. Reversal of the arc from y to x without first reversing the arc from y to z removes the valid status from an Influence Diagram (Figure 1). It will also remove the valid casualty reasoned. Arcs to and from decision nodes cannot be reversed. This arises from the informational nature of the arc to a decision node and the causal nature of the arcs from decision nodes to state nodes (Agogino, 1999).

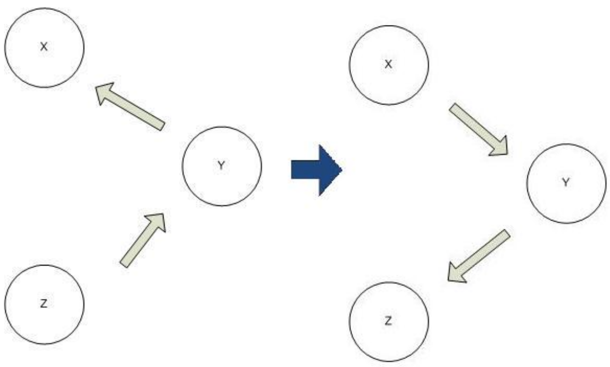

Fig. 2: Reversal Technique

Reversal of the former would violate chronological precedence and that of the latter, causality. According to figure 2, the reversal is only validated when nodes 'X' and 'Y' are conditionally independent.

Shachter furthermore introduced another technique which is known as node removal. Node removal as an algorithm is different from 'barren node removal'; a node in an ID is barren when there is no successor for that node (Shachter R. , 1990). In expert systems the barren node removing mechanism refers to elimination of the nodes which are not relevant to the main decision question. Therefore existence or removal of a barren node in an expert system would not affect the final reasoning in the network (Agogino, 1999). There is an important distinction between the barren node removal in an expert system and in an Influence Diagram. Considering a reversed arc in an ID may make one node barren where there is no successor for the node. This barren node can be removed. Recurrence of this technique would make a less complex ID. Therefore what leaves would be lead to inference in the diagram due to less complexity. Therefore all the remaining nodes can be inferred easily and solve the decision question.

Node removal in an ID classically is summarizing the status and attributes of number of nodes. For instance having the node 'X' with the predecessors of P(X) which are direct predecessors, and proceeds only into the node 'Y' should be removed at the time of evaluations. P(X) becomes direct predecessors of node 'Y'. The status and attributes of 'X' are then captivated in 'Y' (Figure 3).

The removal procedure in an ID involves changes in related mathematical functions.

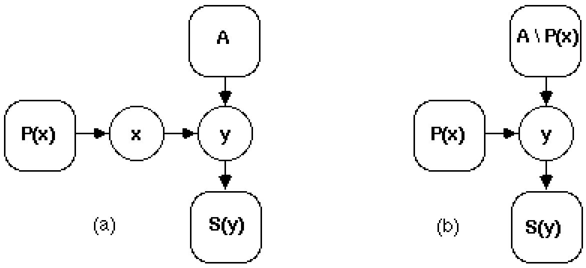

Fig. 3: Removing node 'X' in ID (a) (Agogino, 1999)

As stated earlier the changes in mathematical functions which correspond to the ID in figure 3 (a) are as follows:

$$
\begin{aligned}
\Pr(y \mid P(x), A) &= \sum_{\Omega_x} \Pr(y, x \mid P(x), A, H) \\
&= \sum_{\Omega_x} \Pr(y \mid x, P(x), A) \cdot \Pr(x \mid P(x), A) \\
&= \sum_{\Omega_x} \Pr(y \mid x, A) \cdot \Pr(x \mid P(x))
\end{aligned}
$$

Where Ω_x is the 'X' sample space and Pr represents probability distribution.

This node removal is also referred to as state node removal. There is another situation where the decision node can be removed and that is when the decision node is followed by a value node and succeeds by predecessors of the same value node or variable. This can be used when maximizing one value node in an algorithm. Maximization technique does not consider the probability distribution for the value node, but considers the maximum possible value assigned to it; for instance considering one value node which has a probability to happen would 100% happen.

3. Node Removal-Influence Diagram Case Study: London Plan-Policy 4A.x

The case study selected for this research is the London Plan Policy 4A.2. London Plan is the London mayor's strategic plan authorized by Greater London Authority (GLA). The aim of this policy which is named 'Mitigating Climate Change' is to reduce London CO2 footprints by 60% compared to 1990 base. Figure 5 outlines the interim review:

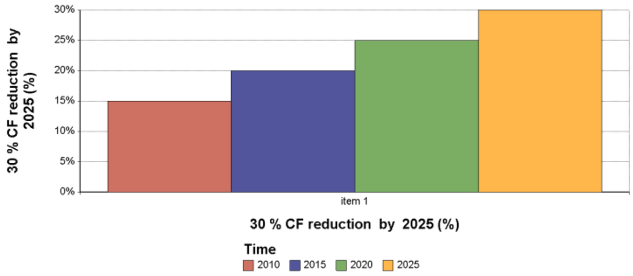

Fig. 4: 30% CF reduction by 2025/60% by 2050 (Hosseinian Far *et al* 2011)

The policy plan proposes the use of Renewable energy sources, Decentralised energy production i.e. Combined Heat and Power (CHP), and utilising less energy by making the energy consumption more efficient (Greater London Authority, 2011).

Considering the general acyclic influence diagrams for the London Plan policy 4A.x (Figure 5), the decision node removal can be placed in the ID:

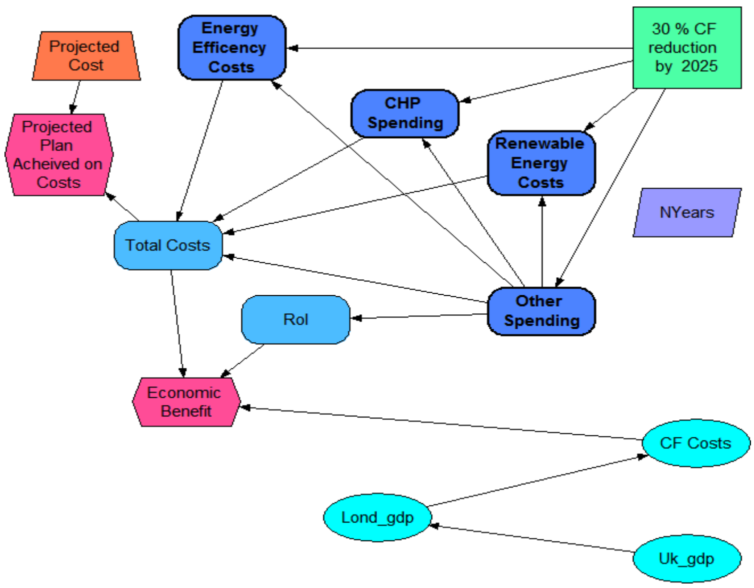

Fig. 5: High Level Influence Diagram for policy 4A.x ID

The main and only decision node in the ID is 30% Carbon Footprint (CF) reduction by 2025. This decision node is preceded by four different variables which are sub-divided into four other influence diagrams.

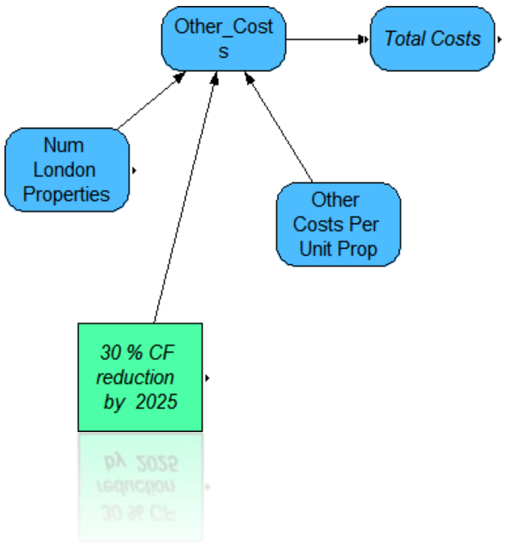

Fig. 6: One of the four variables succeeded by the decision node

There is no successor for this decision node and therefore the decision node removal cannot be considered for evaluating this ID. The reversal and node removal including barren node removal for this influence diagram is still under consideration.

4. Solving Influence Diagrams

There are two different types of solution for an influence diagram. The solution can be either probabilistic inference or decision making under uncertainty. For policy 4A.x ID probabilistic inference is chosen for solving the ID. This involves considering the goal nodes or creating a goal node and performing a reversal operation on the newly created influence diagram.

Before inclusion of the goal node into the solution procedure we can infer the followings:

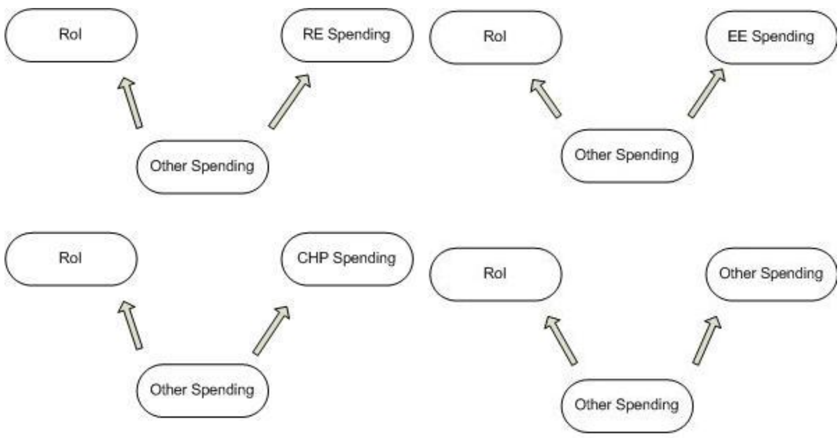

Fig. 7: Variables with 'Other Spending' successor are marginally statistically dependent

Connections in Figure 6 are known as diverging connections.

The connections show that the child nodes are statistically dependent to each other. There are other connections in the ID which show dependence between nodes:

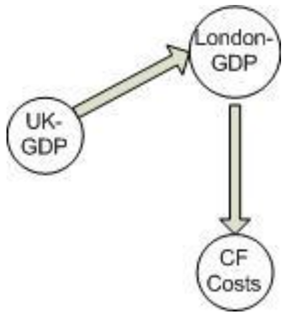

Fig. 8: UK GDP is statistically dependent to CF Costs node

This type of reasoning is known as Casual reasoning where the UK-GDP has statistically effects on the London GDP and that continues to all other proceeded nodes (Hosseinian-Far et al 2011).

On the other hand in figure 8 connections in the parent nodes are not dependent to each other. This reasoning is also referred to as inductive reasoning. The Total Costs are believed to be driven from the individual costs in the parent nodes. There is one type of reasoning known as Explaining Away. Knowing "A"→"B"← "C". The cause of 'C' over 'B' will distract the reader from the cause of 'A' over 'B'. This means an increase in the degree of membership in A will result in a decrease in the degree of membership in B (Hosseinian-Far et al 2011). This type of reasoning does not exist in the current converging connections of our Extended Influence Diagram. But there might be some when the reversal procedure takes place.

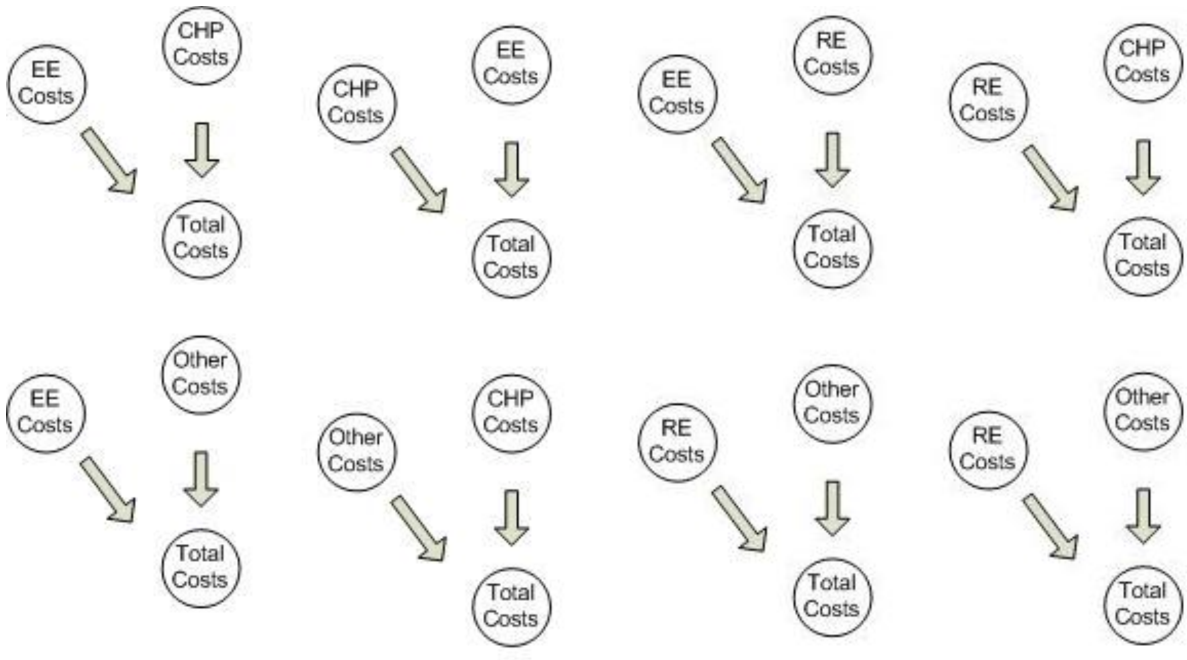

Fig. 9: Parent Nodes are not marginally statistically dependent to each other

Figure 9 illustrates some of the main converging connections in the policy ID.

5. Efficiency Analysis

A brief summary of your research results should be included in this section toward the end of the paper. There are three criteria that should be considered after solving an ID. Accuracy, complexity and efficiency are these three criteria. Calculating the efficiency of the developed ID is also crucial. $\mathcal{E}=\hat{A}^{\alpha}.\ \hat{C}^{1-\alpha}$ is the efficiency function for an ID. The parameters involved are Accuracy (A), and Complexity (C) (Cobb, 2008). In addition, α is the value set by the decision maker. This formula should be looked at with caution, as the complexity and accuracy of the ID should be normalized first (Baye, 2006). According to Cobb (2007) the accuracy and complexity functions are as follows:

$$\hat{\mathcal{A}} = \mathcal{N}_{min} + \frac{(\mathcal{N}_{max} - \mathcal{N}_{min}) \cdot (\mathcal{A} - \underline{\mathcal{A}})}{\overline{\mathcal{A}} - \underline{\mathcal{A}}}$$

$$\hat{\mathcal{C}} = \mathcal{N}_{min} + \frac{(\mathcal{N}_{max} - \mathcal{N}_{min}) \cdot (\mathcal{C} - \underline{\mathcal{C}})}{\overline{\mathcal{C}} - \underline{\mathcal{C}}}$$

A represents accuracy and C represents complexity. The consideration is that when the accuracy is maximized, the complexity of minimized and vice versa. Nmin represents 1 and Nmax represents 2 in Cobb’s research. The Accuracy and Complexity is scaled between a minimum and a maximum in order to assess a trade-off. The complexity of the ID expressions can be calculated using LeafCount function of the Mathematica IDE. Mathematics is software developed by Wolfram which can evaluate the complexity of functions and graphs. The calculation is simple and is counting the variables of any approximation within the ID. It does not only count the variables, but also the expressions defining the function. Therefore LeafCount is a function which counts the number of words, variables and constants in an expression (Wolfram, 2003). The rationale behind this calculation is consideration of the memory required to process the function. The Complexity of an ID then is the summation of all individual complexities:

$$\mathcal{C} = \sum_{i=0}^{n} \mathcal{C}_i = \sum_{i=0}^{n} \sum_{j=1}^{m_i} \mathcal{L}\{\phi_{ij}\}$$

Using the LeafCount function we can simply determine the complexity of expressions and sub-expressions of the London Plan Policy 4A.x. For instance:

```
In[1]:=LeafCount [399943*Num_london_propertie]
Out[1]=3
In[2]:=Level [399943 + * + Num_london_propertie, Heads → True]
Out[2]= {Multiply, 399943, Num_london_propertie}
```

The complexity of the RoI node expression is 3. That is summed up to complexity outputs from the LeafCount function in Mathematica on other nodes. If there are combinatory expressions in the ID, then the NestList function of Mathematica lists all the expression in combinations and then the LeafCount can be run.

For finding Accuracy of Influence Diagram accuracy, the mean squared error between the analytical decision rule and the ID decision rule should be calculated.

The found Accuracy and Complexity values are then traded off using the stated formulas. Cobb’s minimum and Maximum (N) and be scaled using other values, but the technique remains the same. The efficiency of an ID can also be affected by the decision makers’ amendments using α variable.

5. Conclusion

Validation and solution of Influence Diagram requires number of steps to follow. Initially the influence diagram should evaluated using an appropriate algorithm or technique. Once the ID is evaluated, then it should be solved through either probabilistic inference or decision under uncertainty. The London Plan policy ID is under evaluation using reversal and barren reversal techniques and node removals where appropriate. Its evaluation can also be tested using other available algorithms.

Accuracy is calculated using the mean squared error between the final decision rule which is determined using the ID and relative analytical decision rules. Complexity is calculated by the size of the potentials in the general ID, and after each subsequent operation for solving the ID including reversals and removals. Although removal of nodes would reduce the complexity, it can still be measure quantitatively. The efficiency measurement combining accuracy and complexity would become a trade-off that the decision maker or the user would be set.

Further works for this research is completing the reversal and removal technique on the Policy 4A.x ID. Furthermore, the sum of Complexities, the Accuracy and resulting Efficiency is to be completed using the mentioned technique after the evaluation process.

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