A review on non-linear unsteady one dimensional flow in coarse porous media

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Abstract

In this paper, we describe the flow through porous media with emphasis on coarse granular media. A one-dimensional empirical model for single phase Newtonian fluid flow in porous media was introduced by Darcy law. It has long been recognized that the flow through coarse granular media cannot adequately be described by the Darcy Law. The quadratic and power lows are two typical formulations that can be used to extend the Darcy law to non-Darcy flows through coarse porous media. This study of flow through media consisting of large-sized grains is important in a number of civil engineering applications especially Rock fill dams which are usually used for affecting the flood and reducing the peak discharge. In this study, the materials properties for the fluid and porous medium (porosity, viscosity, coarse sieve analysis) can be specified. We employing the constitutive theory, a nonlinear relationship is developed for a Forchheimer equation and finally a generalized from of Forchheimer equation is derived.

Key words: : non-linear, unsteady flow, Forchheimer, non-Darcy flow, coarse porous media.

1. Introduction

Investigation of single-phase fluid flow in porous media is to characterize the system in terms of Darcy's law [1], which assumes that a global index, the permeability K, relates the average fluid velocity U through the process with the head drop, $\Delta\varphi$, measured across the system:

$$V = -K \frac{\Delta\varphi}{\Delta x} \qquad (1)$$

Darcy's law is an empirical law, which states that the flow rate in a porous media is proportional to the pressure gradient in the medium.

Over the years, Darcy's law has been extended to more generalized forms in order to describe more complex flow situations. In addition, it has been observed that the proportionality between fluid velocity and pressure gradient does not hold for high velocity of fluid flow and also in coarse porous media. In high fluid velocity Darcy's law is not valid, hence one has to add some correction terms to the basic Darcy's law in order to take into account the effect of the deviations. This phenomenon has been the subject of many experimental and theoretical investigations. These studies have been centered upon two important issues:

Establishing an upper bound for the range of validity of Darcy's law and providing relationships which predict the nonlinear flow behavior properly

Providing a physical basis for the generalized equation of motion and identifying mechanisms which are responsible for the nonlinear flow behavior.

The purpose of this paper is to review recent research into the non-linear unsteady flow in coarse porous media.

2. Previous works on nonlinear flow in coarse porous media

Many laboratory and numerical studies have been devoted to the determination of the upper range of validity of Darcy's equation. Customarily, this limitation has been signified by means of a critical value for the Reynolds number (defined by $Re = \frac{V.d_m}{n.\upsilon}$) beyond which the head gradient is no more proportional to the flow velocity. Critical values of Reynolds number at the onset of nonlinear flow, according to the most experimental works, range between 1 and 15. Typical values given by various investigators are 1 by Tek (1957), 2 by Wright (1968), 5 by Devries (1979) and 1 to 10 by Dybbs and Edwards (1984). Also, numerical experiments consisting of the solution of Navier-Stokes equations in the pore space of an idealized porous medium give values of Reynolds number between 5 and 13 (e.g., Stark(1972), and Couland at al.(1986)).

Many researchers have developed nonlinear relationships between the flow velocity and the head gradient following different approaches. Various classes of approaches may be identified and are discussed here. More extensive reviews of the subject exist in the works of Bear(1972), Schiedegger (1974), and Hannoura and Barends(1981).

Early theories of high velocity flow could be classified within this group of models. The first equation of motion to account for nonlinear effects was proposed by Forchheimer (Bear, 1972) who suggested the following one-dimensional forms:

$$-\frac{d\varphi}{dx} = aV + bV^2 \tag{2}$$

$$-\frac{d\varphi}{dx} = aV + bV^2 + cV^3 \tag{3}$$

Where φ is the piezometric head, x is the distance in the direction of the flow, V is the magnitude of the flow velocity, and a, b and c are constants. A generalization of Equation 2 to including the unsteady-state effects was proposed by Polubarinova-Kochina (1952):

$$-\frac{d\varphi}{dx} = aV + bV^2 + d\frac{dV}{dt} \tag{4}$$

Where d and t are a constant and time, respectively. Due to the small effect of the unsteady term, Equation 4 has not been used in the literature as a general non-linear equation (Scheidegger, 1974).

Another commonly used non-linear equation is the Missbach equation (Scheidegger, 1974):

$$-\frac{d\varphi}{dx} = rV^s \tag{5}$$

Where r and s are constants which depend on media and fluid properties. s is variable between 1 and 2 and changes from case to case. Although Equation 4 has been widely used in the literature, there is no theoretical basis behind it. It has the advantage to contain only one term and thus better suits the analytical solution of some field problems.

A number of empirical correlations have been given between the Reynolds number and a friction factor accounting for the resistance of a porous medium to the flow of fluid. For example, using this approach Sunada (1965) obtained a one-dimensional non-linear flow equation similar to eq. (2).

Some theoretical derivations of eq. (2) are based on averaging the Navier-Stokes equation. For example, Irmay (1958) averages the Navier-Stokes equation for a model of spheres of equal diameters representing a homogeneous isotropic medium. Employing certain assumptions, he arrives at corresponding equation with a, b in terms of the fluid and medium properties such as viscosity, density, grain size and shape factor. A similar approach has been adopted by Ahmed and Sunada (1969) and Dullien and Azzam (1973). Also, in a recent article Cvetkovic (1986) averages the general form of the linear momentum balance to obtain a macroscopic equation of motion.

Still some other theories of nonlinear flow are based on an idealized geometrical description of a porous medium simple enough to allow the governing continuum equations to be solved. An example of this type of model is that of Blick (1966) which represents a porous medium as a bundle of parallel capillary tubes filled with fluid and with orifice plates spaced along each tube. A static balance of forces is applied to obtain an equation of the Forchheimer's type when the medium is assumed rigid and the fluid is considered to be homogeneous and Newtonian. Although, deviations

Darcy Eq.

Fig.2: Situation of piezometers from each other

Fig. 1: the experimental apparatus

from Darcy's law have been observed at Reynolds numbers of order 10, experiments have indicated that the onset of turbulence occurs at much higher velocities. For example, values up to 300 have been reported (Dybbs and Edwards, 1984).

Another point of view attributes deviations from Darcy's law to the effect of microscopic inertial forces. This point of view has been widely accepted among authors in porous media flow (e.g., Schneebeli, 1955; Bear, 1972; Hubbert, 1956; Scheidegger, 1974; Geertsma, 1974; Happel and Brenner, 1965; MacDonald et al., 1979; Cvetkovic, 1986).

Finally, a third point of view attributes the rise of nonlinear terms to the effects of increased microscopic drag forces on the pore walls (e.g., Firoozabadi and Katz, 1979; Slattery, 1972). The derivation of Forchheimer-type equations presented herein is in supports this point of view.

From the brief discussion of flow models in coarse porous media, it may be deduced that present approaches to this problem are diverse. They are often restricted, through assumptions and constitutive relations, to certain materials and/or special cases (e.g., incompressible, steady-state, one-dimensional, etc.). Thus, their general validity is questionable and, usually, the steps required to extend the results to more general situations are rather obscure. All of them, it seems that a rational and systematic framework within which the mechanics of flow through course porous media could be prove useful.

3-Experimental Setup and materials:

The oscillatory water flume at Hamadan was used for the tests. The length of bottom section equals 13meters. A hydraulic system was applied to force the water through the coarse samples at various periods.

A cross section of the flume equal to $\mathbf{0.6}\times\mathbf{0.6}\boldsymbol{m}^2$ was used. The coarse samples were fixed in the third of the flume. Sample length equaled 2m (Figure1).

The piezometers were mounted approximately 0.20m from each other (figure 2).Types of materials selected in this study were different in size ranging from 2 mm to 19 mm. The mass of the particles was measured to within 0.1 g. The porosity of the materials in the flume was considered to be the average of the porosities of the three related samples.

Darcy Eq.					
V=Ki					
Non-Darcy Eq.					
Forchheimer Equation			Power Equation		
Author(s)	Date	Equation	Author(s)	Date	Equation
Ergun	1952	$i=(\frac{1-n}{n^3})\,[\frac{150\upsilon(1-n)}{gd^2}V+ 1.75\ V^2]$	Wilkins	1956	$i=\frac{1}{m^{0.93}}(\frac{V}{W_n})1.85$
Mc Corquodale	1978	$i=\left(\frac{70\upsilon}{gn\,m^2}\right)V+\left(\frac{0.54\lambda}{gn^{0.5}m}\right)V^2$	Boersma et al.	1973	$i=\left(\frac{Q}{KA}\right)^n, n=(\frac{1}{1-\exp(-1.32r^{0.31})})$
Ergun-Richet	1990	$i=(\frac{1-n}{n^3})[\frac{214M^2v(1-n)}{gd\text{^}2}V+\frac{1.57M}{gd}V^2]$	Stephen	1979	$i=\frac{K_s}{gd\,n^2}V^2$
Carmen	1937	$i=\frac{180(1-n)^2v}{n^3gD^2}V+\frac{2.87(1-n)^{1.1}V^{0.1}}{n^3gD^{1.1}q^{0.1}}V^2$	Martinus	1990	$i=\frac{C_U^{2\alpha}}{2n^2K_M^2ged}V^2$
Englund	1953	$i=780[\frac{(1-n^3)}{n^2}](\frac{v}{gD^2})V+3.6[\frac{1-n}{n^3}](\frac{1}{gD})V^2$			
Muskrat	1937	$i=\frac{\alpha(1-n)^3\mu}{n^2D^2}V+\frac{\beta(1-n)\rho}{n^3D}V^2$			
Irmay	1958	$i=\frac{\alpha(1-n)^2\mu}{D^2(n_w-n)^3}V+\frac{\beta(1-n)}{gD(n_w-n)^3}V^2$			
Scheidegger	1960	$i=\frac{C_1(vT_R^2)}{gn}V+C_2(\frac{T_R^3}{g_n^2})V^2$			
Ward	1964	$i=\frac{v}{gk}V+\frac{0.55}{g\sqrt{k}}V^2$			
Sunada	1965	$i=\frac{v}{gk}V+\frac{CD}{gk}V^2$			
Ahmed	1967	$i=\frac{v}{gk}V+\frac{1}{g\sqrt{ck}}V^2$			
Den Adel	1987	$280[\frac{v(1-n)^2}{gn^3D^2}]V+0.14(\frac{1}{gn^5D})V^2$			

4-Experimental work & test materials

A summary of the key physical properties of materials, the apparatus used, and the experimental procedure applied to measure these properties are introduced. Complete description of the experimental work is found in reference [Shokri, 2004]. Materials had been mechanically sorted in the quarry, it was washed and completely mixed in the laboratory to produce a media as uniform as possible. For example, the size distribution, particle density, porosity were determined. Size distribution of the samples were measured using standard sieve analysis methods and the corresponding d10, d15, d30, d50, d60 and d85 were found for sample. Table 3, summarizes the properties of the sample randomly.

Material	d_{50}(mm)	d_{15}(mm)	Coef. Of Uniformity(-)	Coef. of Concavity(-)	Particle Density (g/cm3)	Porosity(-)
Porous media(I)	8.65	6.05	1.63	1.03	2.57	0.46

Table 3. Material Properties

For the material, the three random samples were tested. The experimental system was constructed in the laboratory of Hamadan University to be able to measure the piezometric head in the direction of the flow in coarse porous medium. Differential heads were measured using manometers.

4-Data Analysis

In this section, the physical properties associated with each empirical equation, reported in Section 3, are applied to the equations to find the hydraulic gradient for all velocity values corresponding to the test.

Piezometers	V(cm/sec)	i(m/m)	$\upsilon(Cm^2 / s)$
Piz1-2	0.0255	0.050	9.36E-3
Piz2-3	0.0257	0.085	9.36E-3
Piz3-4	0.0267	0.105	9.36E-3
Piz4-5	0.0278	0.127	9.36E-3
Piz5-6	0.0293	0.130	9.36E-3
Piz6-7	0.0323	0.165	9.36E-3
Piz7-8	0.0363	0.185	9.36E-3
Piz8-9	0.0411	0.225	9.36E-3
Piz9-10	0.0484	0.300	9.36E-3
Piz10-11	0.0605	0.344	9.36E-3
Piz11-12	0.0686	0.395	9.36E-3

Table 3. Experimental data collected for the test.

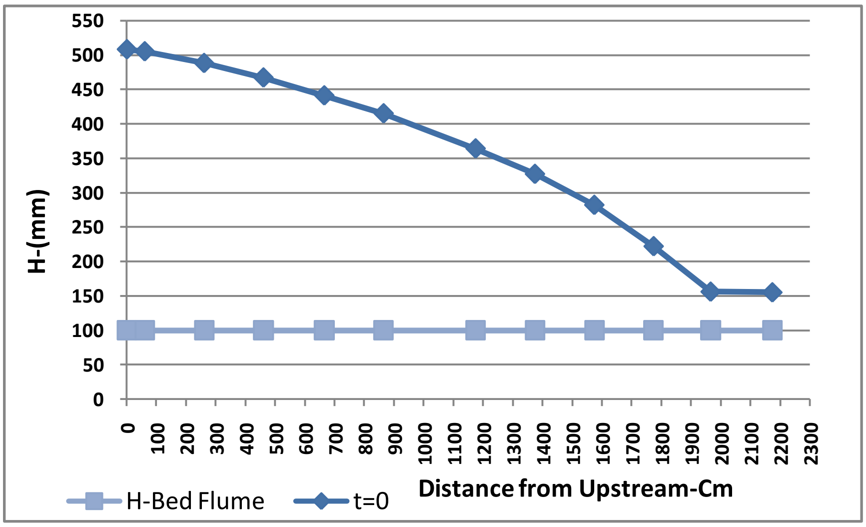
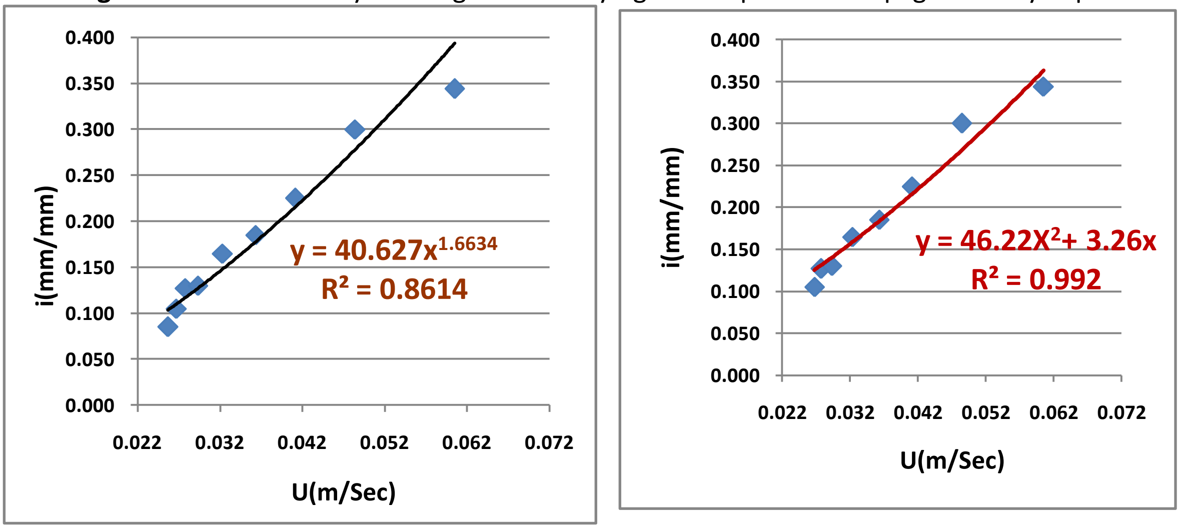


Fig3- Observations of head Variation vs. Distance from upstream

Fig4- Observations of hydraulic gradient varying with superficial seepage velocity in porous media



Material	d_{50} (mm)	Porosity (-)	a (s/m)	b (s^2/m^2)
Small	8.5	0.489 2.77	160.0	
Medium	21.1	0.459	0.99	62.0
Large	27.6	0.443	0.82	39.1

Table 4: Material Properties Used in Averaging Scheme

5- Conclusions

Due to the financial restrictions at the end of the program, it was not possible to perform a comprehensive parametric study. However, tests were planned in order to get a good insight in some of the fundamental problems and to obtain approximate values of the main parameters, which characterize the flow resistance. Despite this, it is clear that a comprehensive parametric study is still needed. In this paper, the empirical equations that estimate hydraulic parameters for non-linear flow through coarse porous media are evaluated using a series of independent data collected in the laboratory. The study shows that McCorquodale et al. and Stephenson equations, which some subjective parameters related to the surface characters of the material, have been incorporated in their structures, can give good results.

The purpose of the current study was to determine and develop a nonlinear relationship between the pressure gradient and the flow velocity. The development is based on physical and mathematical principles and is carried out at the level of observations (the macroscopic level). Using a standard order-of-magnitude argument, microscopic inertial forces have been shown to be small at the onset of nonlinear flow (Re = 10). The nonlinear dependence of interfacial drag forces on the flow velocity has been shown to increase to the nonlinear behavior of flow at high velocities.

The results show that the two coefficients associated with the quadratic law vary differently; the coefficient used for the linear term fluctuates for large Reynolds number and that for nonlinear term varies significantly for low Reynolds number. On the other hand, the two parameters (i.e., a & b) included in the quadratic law vary more significantly and also correlate to each other. This study suggests that the quadratic law generally represents well (but not exactly) the resistance of the seepage flow including the Darcy and non-Darcy regimes, while the power law is applicable only when the seepage velocity varies within a limited range.

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